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AN ANALYSIS OF HEAT TRANSFER TO TURBULENT FLOW OF DRAG REDUCING FLUIDS

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Abstract—Reichardt's analysis for heat transfer to turbulent flow in smooth pipe is extended to drag reducing viscoelastic fluids. Velocity profile expression by Deissler is modified to account for changes brought about by viscoelastic drag reducing fluids. The final correlation compares very favourably over a wide range of Prandtl (or Schmidt) numbers, with experimental data.

NOMENCLATURE

- A, slope of logarithmic velocity profile in equation (9);
- B, intercept parameter defined in equation (9);
- b, integral defined in equation (6);
- C_p, specific heat;
- D, diameter of pipe;
- D_e, Deborah number $\theta u^{*2}/v$;
- E_h, eddy diffusivity for heat transfer;
- E_m, eddy diffusivity for momentum transfer;
- f, fanning friction factor, $\tau_0/\frac{1}{2}\rho V^2$;
- G, constant defined in equation (10), numerical value is 3.0;
- n, constant in equation (8);
- Nu, Nusselt number;
- Pr, Prandtl number, $C_p\mu/k$;
- k, thermal conductivity;
- q, heat flux;
- Re, Reynolds number, DV/v;
- St, Stanton number, $Nu/RePr$;
- T, temperature;
- U, velocity at the center line of the pipe;
- u, time average local axial velocity;
- u^*, friction velocity;
- u^+, dimensionless velocity u/u^*;
- V, average velocity;
- X, ratio of E_h/E_m;
- y, distance from the wall in the radial direction;
- y^+, dimensionless distance yu^*/v.

Greek symbols

- τ_0, shear stress at the wall;
- ρ, density;
- v, kinematic viscosity;
- μ, shear viscosity;
- θ, relaxation time for Maxwell fluid;
- θ_m, ratio of mean to maximum temperature difference;
- ϕ_m, ratio V/U.

INTRODUCTION

DRAG reduction under turbulent flow conditions, has been obtained with very dilute solutions of polymers. Mechanistic interpretations of the phenomena have been centred around viscoelastic properties of these polymer solutions [1–3]. Many important aspects of the phenomena are reported in comprehensive manner by Hoyt [4] and Virk [5]. Heat transfer to these drag reducing polymer solutions is of importance for both theoretical and practical reasons. The heat or mass-transfer results will aid in understanding the fluid mechanics of the turbulent flow. Previous studies on heat transfer have indicated that reduction in heat transfer is much more conspicuous than reduction in momentum transfer [6–8]. However, not many analyses account for the conspicuous reductions in heat transfer and the present paper is directed to this end.

BACKGROUND OF THE PROBLEM

The equations for momentum and heat transfer may be written respectively as:

$$\frac{\tau}{\rho} = (E_m + v)\frac{du}{dy} \tag{1}$$

and

$$\frac{q}{\rho C_p} = (E_h + v/Pr)\frac{dT}{dy}. \tag{2}$$

Several authors have assumed different forms for the variation of the eddy diffusivities with distance from the wall and have given solution for predicting heat-transfer coefficient as a function of Reynolds and Prandtl numbers but one of the most significant analyses is that of Reichardt [9] because of minimal assumptions employed. These are merely that (i) the ratio of eddy diffusivity for heat to that for momentum transfer is constant and (ii) the heat flux varies linearly with radial position. Since details are available elsewhere [10–11] his final result may be stated directly as:

$$St = \frac{f/2 \cdot X \cdot 1/\theta_m}{1/\phi_m + (Pr \cdot X - 1)(f/2)^{1/2}\int_0^{U/u_*} \frac{du^+}{1 + XPrE_m/v}}. \tag{3}$$

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The integral in the denominator of the above equation is denoted as "b" by Reichardt and the same notation will be followed here. Further, if one assumes:

$$X = E_h/E_m = 1.0 \tag{4a}$$

$$\theta_m = 1.0 \tag{4b}$$

one obtains:

$$St = \frac{f/2}{1/\phi_m + (Pr-1)(f/2)^{1/2}b} \tag{5}$$

in which

$$b = \int_0^{U/u_*} \frac{du^+}{1+Pr\cdot E_m/\nu}. \tag{6}$$

Friend and Metzner [10] determined this "b" function experimentally for purely viscous fluids and correlated it as

$$b = 11.8(Pr)^{-1/3} \tag{7}$$

and also suggested that the average ratio of maximum to mean velocity over a wide range of Reynolds number may be taken as 1.2.

Inspection of equation (6) would reveal that this "b" function can be evaluated if accurate information on either the velocity profile or equivalently on the variation of the eddy diffusivity with distance from the wall is available. Many expressions for velocity profile for Newtonian fluids exist but most of them contain more than one adjustable parameter and this renders them useless for systems differing from Newtonian behaviour. However, the form of the velocity profile suggested by Deissler [11] is free from such difficulties and one single form describes the velocity profile for both the viscous sublayer and the transition zone. This form of velocity profile together with a semi logarithmic form for the turbulent core, can thus describe the entire field.

ANALYSIS

Deissler suggested the following expression for the velocity near the wall:

$$u^+ = \int_0^{y^+} \frac{dy^+}{1+n^2u^+y^+(1-e^{-n^2u^+y^+})}. \tag{8}$$

The velocity profile in the core region can be approximated in general form as:

$$u^+ = A\ln y^+ + B. \tag{9}$$

Equation (8) approaches equation (9) at a value of y^+ dependent upon n. Deissler found that with $n = 0.124$, equation (8) described the velocity profile in the region $0 \leqslant y^+ \leqslant 26$. For the turbulent core ($y^+ > 26$) he assigned the value of 2.76 for the parameter A in equation (9). However, solutions of equation (8) for various values of n may be used to show that $n = 0.127$ described the velocity profile in the wall region upto $y^+ = 40$ and enables use of equation (9) for the turbulent core with $A = 2.5$ and $B = 5.0$, the conventional values of these parameters.

Equation (8) suggests that if the constant n decreases, the eddy diffusivity will also decrease. Recently,

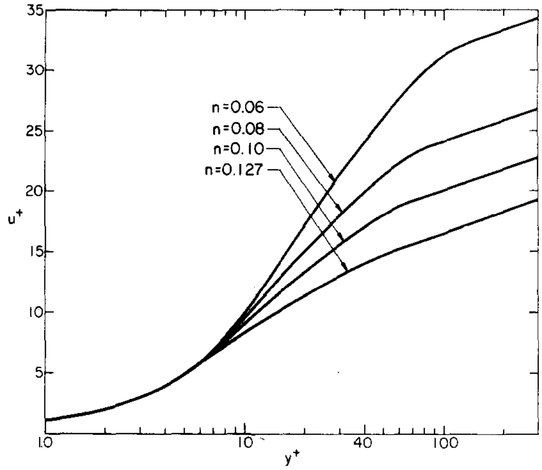

FIG. 1. Velocity profiles in the wall region as a function of the Diessler parameter n.

Debrule [12] found that his heat-transfer results for Polyox solutions were correlated using values of n much below that for Newtonian fluids. Equation (8) was therefore solved for various values of n and the resulting velocity profiles are plotted in Fig. 1. It is seen that with decreases in n, the velocity profile in the wall region approaches that of the core at increasing values of y^+ and that the intercept B in equation (9) increases as this is done. For drag reducing fluids, the core region velocity profile has the same slope as that for Newtonian fluids but with higher values of intercept [13–16]. Carrying this further, the function b in equation (6) was evaluated for different values of n using the velocity distribution from equation (8). Table 1 lists the values of n with the corresponding values of the parameter B, Reichardt function b, and values

Table 1. Influence of n on the parameters B, b and y^+_{cr}

n	B	b at $Pr = 1000$	y^+_{cr}
0.127	5.0	1.571	40
0.100	8.5	1.996	54
0.080	12.5	2.490	70
0.060	20.0	3.320	110

of y^+_{cr} at which velocity profile from the core region smoothly joins with equation (8). It is seen that the function b increases as n decreases. Increases in b over the value for Newtonian fluids clearly means that the heat-transfer coefficient will further be reduced which is in fact in agreement with the observed results of many investigators [6–8, 13, 17, 18].

The relationship between the friction factor and Reynolds number, from the velocity profile, for the turbulent flow can be approximated as:

$$\frac{1}{\sqrt{f}} = \frac{A\ln(Re\sqrt{f}) + B - A\ln(2\sqrt{2}) - G}{\sqrt{2}}. \tag{10}$$

Seyer and Metzner [16] evaluated the parameter B for drag reducing fluids from velocity profile measurements as well as from pressure drop data. They also measured the rheological properties of the same fluids and correlated the increase in B with viscoelastic properties of drag reducing fluids through a Deborah number defined as $\theta u^{*2}/\nu$. Thus, for each value of n in equation (8), a corresponding value of the Deborah number could be assigned through this relationship. Thus, the increases in "b" function can be correlated with corresponding Deborah number (De). For this purpose, it is decided that the function b could be split as:

$$b = b_0 + b_1 \tag{11}$$

in which b_0 is the obtained value for Newtonian fluids with $n = 0.127$. Obviously, b_0 is a function of Prandtl number only. Solving equation (6) numerically for different Prandtl numbers, it is possible to plot the values of b against Pr and approximate as

$$b_0 = 9.2(Pr)^{-0.258}. \tag{12}$$

The increase in $b(b_1)$ is due to the presence of Deborah number. Equation (6) was numerically solved for different values of n. Noting that each value of n corresponds to a Deborah number as discussed earlier, it is possible to obtain function b and therefore b_1 by subtracting b_0 from b. The results from these computations can be plotted as a function of Prandtl number for a fixed value of Deborah number (fixed chosen value of n) and also as a function of Deborah number (different values of n) for a fixed Prandtl number. Thus, the computed results may be approximated as

$$b_1 = 1.2(De)(Pr)^{-0.236}. \tag{13}$$

Combining equations (5) and (11)–(13), the final result for the Nusselt number may be written as:

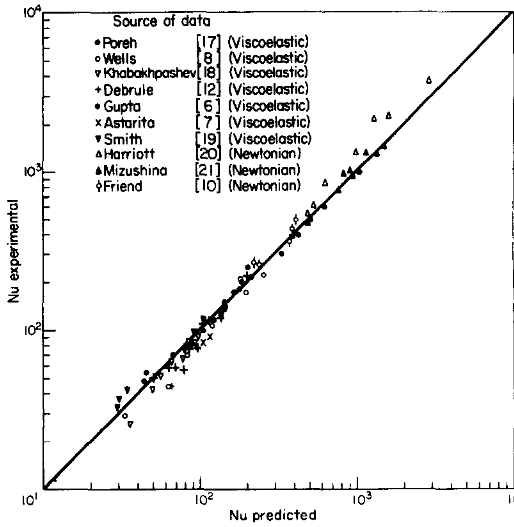

FIG. 2. Comparison of experimental Nusselt numbers with predictions from equation (14).

be safely done as shown by Kale [22]. Kale has satisfactorily correlated drag reduction data for rotating disc using polyethylene oxide (WSR 301) solutions. In the absence of actual friction factor data, either the knowledge of viscoelastic properties given by fluid relaxation time as a function of shear stress or measurement of friction factor under turbulent flow conditions at any two flow rates (i.e. wall shear stress) is sufficient to estimate the Deborah number as suggested by Seyer and Metzner. As seen from Fig. 2, the predicted values from equation (14) are in good agreement with the experimental observations. Recently Dimant and Porch [23] have shown that the

$$Nu = \frac{f/2 \cdot Re \cdot Pr}{1.2 + (Pr-1)(f/2)^{1/2}\{9.2(Pr)^{-0.258} + 1.2(De)(Pr)^{-0.236}\}}. \tag{14}$$

In this equation, the parameter $1/\phi_m$ has been retained at its Newtonian value of 1.2 because it will be close to this value for dilute polymer solutions of primary interest. Reduction in friction factor approaches the asymptote for maximum reduction for values of Deborah number $\geqslant 20$. Therefore, it may be possible to write the asymptotic equation for Nusselt number as:

$$Nu = \frac{f/2 \cdot Re \cdot Pr}{1.2 + (Pr-1)(f/2)^{1/2} \cdot \{9.2(Pr)^{-0.256} + 24(Pr)^{-0.236}}. \tag{15}$$

COMPARISON WITH DATA

Data from various sources [6–8, 12, 17–21] on heat and mass transfer are analysed and compared with predictions from equation (14). It is realized that equation (14) is, strictly speaking, for isothermal cases. Therefore, the experimental data showing small temperature changes only are considered for comparison's sake. The Deborah numbers are calculated from actual friction factor data reported. Using equation (10), the function B is calculated. Corresponding Deborah number was evaluated using graphical relationship given by Seyer and Metzner [16]. Although Seyer and Metzner established this relationship using polyacrylamide solutions, its extension to other polymers can thermal entry lengths for viscoelastic fluids are much larger compared to Newtonian fluids. The data suspected to be in the thermal entry zone are not considered for the comparison with equation (14).

The functional form of b for Newtonian fluids in equation (12) is different from that given by equation (7). The accuracy of the chosen form for the velocity profile, will play a significant role in the final form for b and the data at extremely high Prandtl numbers free from the effects due to roughness of the pipe can only decide the accuracy of above predictions for Newtonian fluids.

For Newtonian fluids, the data of Mizushina *et al.* [21] show excellent agreement while data from Harriott and Hamilton [20] are consistently higher than predicted. The experimental technique used by the latter of the above two sources could have led to slight roughness at the dissolving wall and could have significant effect in enhancing mass transfer particularly at high Schmidt numbers. The electrolytic technique used by Mizushina *et al.* is free from such a possibility. Their data compare more accurately with the predictions from equation (14). Recently, Hughmark [24] has developed a correlation from available data on heat and mass transfer to Newtonian fluids, valid over a much wider range of Prandtl number. [$0.2 \leqslant Pr \leqslant 100\,000$]. His predictions compare very favourably with equation (14) for Newtonian fluids.

At this point, a comparison of equation (14) could be made with other correlations. Most of the workers have used different empirical relationships for variation of eddy diffusivity with the distance from the wall. The usual form of these relationships is:

$$\frac{E_m}{\nu} = a(y^+)^{b'}. \tag{16}$$

Different values of b' have been used. Correlating the experimental data, Friend and Metzner have indicated $b' = 3.0$. Similarly, Hughmark [25] and Wasan *et al.* [26] have shown that $b' = 3.0$ is more consistent with experimental observations. Son and Hanratty [27] have, however, shown that limiting value of b' close to the wall is 4.0. Equation (14) agrees with this value.

Equation (14) is, as noted earlier, for isothermal conditions. The Sieder–Tate type correction for non isothermal conditions may be used but no attempt was done to test this due to lack of sufficient and accurate data.

CONCLUSIONS

Equation (14) covers a wide range of Prandtl number in comparison with many other available correlations.

Analogous mass-transfer problem is of importance when drag reducing additive is used in the boundary layer only: it helps in determining how quickly it will be transported away.

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ANALYSE DE LA CONVECTION THERMIQUE TURBULENTE DANS LES FLUIDES A REDUCTION DE TRAINEE

Résumé—L'analyse par Reichardt de la convection thermique turbulente dans les tubes lisses est étendue aux fluides viscoélastiques réducteurs de trainée. L'expression donnée par Deissler pour le profil des vitesses est modifiée pour tenir compte des changements apportés par ces fluides. La formule finale s'accorde favorablement, pour un grand domaine du nombre de Prandtl (ou de Schmidt), avec les résultats expérimentaux.

EINE UNTERSUCHUNG DES WÄRMEÜBERGANGS BEI TURBULENTER STRÖMUNG WIDERSTANDSMINDERNDER FLUIDE

Zusammenfassung—Die von Reichardt aufgestellte Theorie über den Wärmeübergang an turbulente Strömungen in glatten Rohren wird auf widerstandsmindernde, viskoelastische Fluide ausgedehnt. Der Deissler-Ansatz für das Geschwindigkeitsprofil wird so modifiziert, daß er den durch widerstandsmindernde, viskoelastische Fluide verursachten Abweichungen Rechnung trägt. Für einen großen Bereich von $Pr(Sc)$-Zahlen stimmt die schließlich gefundene Korrelation gut mit Versuchsdaten überein.

АНАЛИЗ ПЕРЕНОСА ТЕПЛА К ТУРБУЛЕНТНОМУ ПОТОКУ ЖИДКОСТИ С ПОЛИМЕРНЫМИ ДОБАВКАМИ

Аннотация — Метод Рейхардта для исследования переноса тепла в трубе с гладкими стенками распространен на анализ уменьшения сопротивления при течении вязкоупругих жидкостей. Предложенный Дайслером профиль скорости модифицирован с целью учета эффекта уменьшения сопротивления за счет вязкоупругих сил. Полученное соотношение дает хорошее согласие с экспериментальными данными в широком диапазоне чисел Прандтля (Шмидта).